Effect of Dissipative Phenomena on the Evolution of Shock Waves

V. P. Shidlovsky*
Academy of Sciences of the USSR, Moscow, USSR

The Navier-Stokes equations for unsteady one-dimensional motion of gas having a perturbation front are reduced to a special dimensionless form. At small values of the argument responsible for the influence of dissipation, there are singular zones adjacent to: 1) the perturbation front, and 2) the source of generation of the motion. The simplified equations governing motion within corresponding boundary layers consequently are derived. A finite relation indicates whether, for given parameters of the viscous compressible flow, the perturbation front degenerates with time into a shock discontinuity, or viscous perturbations are going to spread throughout the whole flow. A sample numerical analysis of the complete Navier-Stokes equations is conducted for the case of a point explosion.

Nomenclature

A = factor in viscosity law
B = factor in the law of variation of unperturbed gas density
e = internal energy
K = function of χ [Eq. (10)]
n = exponent in the viscosity law
N = dimensionless temperature
p = pressure
P = dimensionless pressure
r = spatial coordinate
r_f = perturbation front coordinate
r_p = source of perturbation coordinate
r_s = shock front coordinate
R = dimensionless density
t = time
T = temperature
U = dr_f/dt
v = velocity
V = dimensionless velocity
Z = function of χ [Eq. (7)]
κ = specific heat ratio
η = r/r_f
μ = viscosity
ν = integer specifying type of symmetry
ρ = density
σ = Prandtl number
χ = dimensionless argument [Eq. (5)]
ψ = dimensionless total energy [Eq. (8)]
ω = exponent in the law of variation of unperturbed gas density

Introduction

WITHIN the framework of the classical theory of unsteady one-dimensional motion of gas with shock waves (see Ref. 1), solutions obtained without considering dissipative phenomena sometimes lead to physically unrealistic results. Thus, the self-similar solution of the point-blast problem produces infinite temperature and zero density at the center, independent of the time elapsed since the beginning of the process. In order to obtain the actual temperature at the center of the blast, it is necessary to account for dissipative heat transfer,[2-4] whereas to exhibit the structure of the blast-wave front, one should also take into consideration viscous effects.[5]

Received June 26, 1975.
Index categories: Nonsteady Aerodynamics; Shock Waves and Detonations; Viscous Nonboundary-Layer Flows.
*Senior Research Scientist, Computing Center.

It is usually assumed that by explosion-like processes the perturbation front of a viscous and heat-conducting gas is transformed into a shock wave, as a rule, within a negligibly small time interval (of the order of the molecular relaxation time). After that period, the main characteristic features of the phenomenon are described quite adequately by the self-similar theory of adiabatic motions, whereas the singularity at the center might be accounted for by means of the simplest correction. This kind of flow evolution under the influence of dissipation is revealed in a number of examples, provided that the density of the unperturbed gas is constant, and generates the widespread view that the effects of dissipation on unsteady gas flows are only momentary. The aim of the present work is to check this thesis by means of analysis of the complete system of Navier-Stokes equations, with an arbitrary power law relating viscosity and heat conductivity to temperature, and with a power-law variation of the unperturbed gas density.

Transformation of the Navier-Stokes Equations

Let a real perfect gas with constant specific heat ratio κ and Prandtl number σ be in a state of unsteady one-dimensional motion. the Navier-Stokes equations for such a motion have the form

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial v}{\partial r} + \frac{\nu - 1}{r} v\right) = 0$$

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r}\right) + \frac{\partial p}{\partial r} = \frac{4}{3}\frac{\partial}{\partial r}\left[\mu\left(\frac{\partial v}{\partial r} - \frac{\nu - 1}{2}\frac{v}{r}\right)\right] + 2(\nu - 1)\frac{\mu}{r}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right) \tag{1}$$

$$\rho\left(\frac{\partial e}{\partial t} + v\frac{\partial e}{\partial r}\right) + p\left(\frac{\partial v}{\partial r} + \frac{\nu - 1}{r} v\right) = \frac{\kappa}{\sigma} r^{1-\nu}\frac{\partial}{\partial r}\left(r^{\nu - 1}\mu\frac{\partial e}{\partial r}\right) + 2\mu\left\{\left(\frac{\partial v}{\partial r}\right)^2 + (\nu - 1)\frac{v^2}{r^2} - \frac{1}{3}\left[\frac{\partial v}{\partial r} + (\nu - 1)\frac{v}{r}\right]^2\right\}$$

$$p = (\kappa - 1)\rho e \qquad \mu = Ae^{\eta} \quad n \quad (A, n = \text{const})$$

Here the integer ν specifies the type of symmetry of motion and equals 1 for planar s, 2 for cylindrical, and 3 for sphericai symmetry.

Assume that the unperturbed gas density is distributed in space according to a law

$$\rho_0 = Br^{-\omega} \tag{2}$$

with positive constants B and ω. Moreover, the unperturbed gas will be supposed at rest with zero values of temperature and pressure. Hence, the region of the perturbed motion may be thought of as having finite extent and being bounded by the surface $r=r_f(t)$, i.e., by the perturbation front. The law of motion of the front $r_f(t)$, is determined in the process of solution.

Of course, the form of the initial and boundary conditions depends on the nature of the problem. For the case of the blast problem, we have

$$\rho=\rho_0 \quad p=e=v=0 \quad (\kappa/\sigma)\mu(\partial e/\partial r)=\mu(\partial v/\partial r)=0$$
$$\text{at } t=0 \text{ and } t>0, \qquad r\geq r_f \tag{3}$$
$$v=(\kappa/\sigma)\mu(\partial e/\partial r)=0 \text{ at } t>0,\ r=0$$

An additional integral condition follows from conservation in time of the total energy of gas within the perturbed volume

$$[2(\nu-1)\pi+\tfrac{1}{2}(\nu-2)(\nu-3)]\int_0^{r_f}\rho(e+\tfrac{1}{2}v^2)r^{\nu-1}dr$$
$$=E=\text{const} \tag{4}$$

For a number of problems formulated in terms described previously, there exist, in the limit of vanishing viscosity, self-similar solutions (see Ref. 1). When transforming Eqs. (1) to a dimensionless form, it is useful to choose one of the new arguments in such a way that, in the inviscid limit, it reduces to a self-similar variable $\lambda=r/r_s$. Recalling the fact that, as $\mu\to 0$ we have $r_f\to r_s$, we satisfy the aforementioned condition with the variable $\eta=r/r_f$. The choice of the second argument may be subjected to two conditions: 1) it should not depend on the spatial coordinate r; and 2) it should vanish in the limit $\mu\to 0$ because of similarity of the limiting form of motion. With the notation $dr_f/dt=U$, one may express the new arguments, which comply with these demands, in the form

$$\eta=r/r_f \qquad x=\frac{A}{(\kappa-1)^n}\frac{U^{2n-1}}{Br_f^{1-\omega}} \tag{5}$$

During the subsequent analysis, the function $x(t)$ is assumed to be continuous, monotonic, and differentiable. These properties follow from general estimates of the physical character of the motion and of the mathematical character of Eqs. (1), and are confirmed by a subsequent check for a number of cases investigated.

The dependent variables in Eqs. (1) also are transformed to new dimensionless functions of the arguments η and x, according to the relations

$$v=UV(\eta,\chi) \qquad \rho=Br_f^{-\omega}R(\eta,\chi)$$
$$p=Br_f^{-\omega}U^2P(\eta,\chi) \quad e=(\kappa-1)^{-1}U^2N(\eta,\chi) \tag{6}$$
$$\mu=\chi Br_f^{-\omega}UN^n(\eta,\chi)$$

Transformation of Eqs. (1) demands the introduction of a certain function

$$Z(\chi)=(dU/dt)r_fU^{-2} \tag{7}$$

which is not known in advance, but remains finite at all values of the argument χ. Note that the extremal value $Z(0)=Z_0$ is known and corresponds to a self-similar solution.

When analyzing the blast problem, it is worthwhile to introduce a dimensionless function that, for given χ, is proportional to the total gas energy within the perturbed volume

$$\psi(\chi)=\int_0^1 R\left(\frac{1}{\kappa-1}N+\tfrac{1}{2}V^2\right)\eta^{\nu-1}d\eta \tag{8}$$

If one returns to the law of conservation of total energy [Eq. (4)], then, after conversion to dimensionless variables under the integral sign and introduction of a function $\psi(\chi)$, this equation is transformed to the relation

$$(d/d\chi)[r_f^{\nu-\omega}U^2\psi(\chi)]=0$$

or

$$K\chi(d\psi/d\chi)+(2Z+\nu-\omega)\psi=0 \tag{9}$$

where we have introduced the notation

$$K(\chi)=\omega-1+(2n-1)Z(\chi) \tag{10}$$

In the new variables corresponding to Eqs. (5) and (6), Eqs. (1) acquire the form

$$(V-\eta)\frac{\partial R}{\partial\eta}-\omega R+K\chi\frac{\partial R}{\partial\chi}+R\frac{\partial V}{\partial\eta}+\frac{\nu-1}{\eta}RV=0$$

$$R\left[ZV+(V-\eta)\frac{\partial V}{\partial\eta}+K\chi\frac{\partial V}{\partial\chi}\right]+\frac{\partial P}{\partial\eta}$$
$$=\frac{4}{3}\chi\frac{\partial}{\partial\eta}\left[N^n\left(\frac{\partial V}{\partial\eta}-\frac{\nu-1}{2}\frac{V}{\eta}\right)\right]+2(\nu-1)\chi\frac{N^n}{\eta}\left(\frac{\partial V}{\partial\eta}-\frac{V}{\eta}\right)$$

$$R\left[2ZN+(V-\eta)\frac{\partial N}{\partial\eta}+K\chi\frac{\partial N}{\partial\chi}+(\kappa-1)N\left(\frac{\partial V}{\partial\eta}+\frac{(\nu-1)}{\eta}V\right)\right]$$
$$=\frac{\kappa}{\sigma}\chi\eta^{1-\nu}\frac{\partial}{\partial\eta}\left(\eta^{\nu-1}N^n\frac{\partial N}{\partial\eta}\right)+2(\kappa-1)\chi N^n\left\{\left(\frac{\partial V}{\partial\eta}\right)^2\right.$$
$$\left.+(\nu-1)\frac{V^2}{\eta^2}-\frac{1}{3}\left[\frac{\partial V}{\partial\eta}+(\nu-1)\frac{V}{\eta}\right]^2\right\} \qquad P=RN \tag{11}$$

After transformation of dimensionless variables, the conditions of Eqs. (3) become

$$R=1 \qquad P=N=V=0$$
$$\chi N^n\partial N/\partial\eta=\chi N^n\partial V/\partial\eta=0 \qquad \text{at finite } \chi,\ \eta=1$$
$$K\chi d\psi/d\chi+(2Z+\nu-\omega)\psi=0 \tag{12}$$
$$V=\chi N^n\partial N/\partial\eta=0 \text{ at finite } \chi,\ \eta=0$$

Here we have deliberately omitted the initial conditions, that is, the conditions at one of the extreme values of χ within the range considered. The reason is that, independent of the particular form of dependence $\chi(t)$, we shall always prefer to supplement the conditions of Eqs. (12) by the additional requirement that at $\chi=0$ the solution reduces to the self-similar one for adiabatic flow.

Boundary Layers

Despite the fact that the flow structure at $\chi=0$ is known, the transition from zero to an arbitrarily small value of χ corresponds to a singular perturbation, because it is connected with an increase in the order of the equations and with modification of the boundary conditions. At small values of χ, the vicinities of the perturbation front and of the source of perturbations (in the present case, the center of the blast) play the role of boundary layers. The usual technique (see Ref. 6) is used for investigation of these boundary layers.

The boundary layer at the perturbation front is actually concurrent with the shock-wave structure. There are many papers devoted to the study of shock-wave structure,[7-12] although the majority of them deal with steady flows. To study the vicinity of a perturbation front with the inner boundary $\eta=1$, we transform η so that the viscous and the main convective terms of Eqs. (11) become of the same order in χ.

This is achieved by setting

$$\eta = 1 - \chi\eta^* \tag{13}$$

The scales of the dependent variables are not changed by the transformation Eq. (13), although we shall use starred symbols for them here. Upon inserting Eq. (13) into Eqs. (11) and passing to the limit $\chi \to 0$, we obtain the simpler equations of the frontal boundary layer. Then we use the condition, valid for the blast problem,

$$Z(0) = Z_0 = \tfrac{1}{2}(\omega - \nu)$$

and lower the order of the equations with the help of the inner boundary conditions following from Eqs. (12). Thus, we get the simple system of ordinary differential equations:

$$R_* = 1/(1 - V_*)$$
$$4/3 N_*^n V_*' = V_* - R_* N_*$$
$$(\kappa/\sigma) N_*^n N_*' = N_* - [(\kappa - 1)/2] V_*^2 \tag{14}$$

where prime denotes the derivative with respect to η_*. The properties of the system of Eqs. (14) are quite similar to those of the equations for the structure of a steady shock wave of arbitrary strength, whereas the equations of the first approximation for a shock wave in hypersonic flow[5] coincide with Eqs. (14), aside from constant normalizing factors.

The conditions of matching with the outer adiabatic flow are derived in accordance with the general principle.[6] For the present case, within the accuracy of exponentially decaying terms, we obtain

$$R_* \to \frac{\kappa - 1}{\kappa - 1} \quad V_* \to \frac{2}{\kappa + 1} \quad N_* \to \frac{2(\kappa - 1)}{(\kappa + 1)^2} \text{ as } \eta_* \to \infty \tag{15}$$

From Eqs. (14) and their boundary conditions, it is evident that the flow of gas within the frontal boundary layer depends neither on the parameter of dimensionality of the problem ν nor on the parameter of variation of unperturbed density ω. The solutions of Eqs. (14) contain a singularity at $\eta_* = 0$, and in order to construct a numerical solution it is necessary to use asymptotic expansions. For the case $n = \tfrac{1}{2}$, $\kappa = 2\sigma$ the first terms of the expansions are

$$N_* = (1/16 \times \eta_*^2 + O(\eta_*^4)$$
$$V_* = (3/16)\eta_*^2 + C\eta_*^3 + O(\eta_*^4)$$
$$R_* = 1 + (3/16)\eta_*^2 + C\eta_*^3 + O(\eta_*^4) \tag{16}$$

The expressions Eqs. (16) provide the satisfaction of all the boundary conditions at $\eta_* = 0$, as well as an approximate satisfaction of Eqs. (14), and by means of an adjustment of the constant C it is possible to achieve the satisfaction of the asymptotic conditions Eqs. (15).

For the case of the blast problem, the source of perturbations is the center of explosion. In order to obtain the equations of the central boundary layer, one resorts to a change of variables

$$V = \tilde{V}\chi^\beta \quad \eta = \tilde{\eta}\chi^\beta \quad N = \tilde{N}\chi^{-\alpha} \quad R = \tilde{R}\chi^\alpha \tag{17}$$

After introducing this transformation into Eqs. (11), we must once again pass to the limit $\chi \to 0$. One of the conditions for determining the constants α and β is that terms in the transformed energy equation, which are responsible for convective heat transfer and for molecular heat conduction, must be of equal order in χ. This gives

$$1 - (n + 1)\alpha = 2\beta \tag{18}$$

The second condition is that matching of the solution for the central boundary layer with the outer flow should occur as $\tilde{\eta} \to \infty$, independently of the fixed value of χ. As is known from the self-similar solution,[1] near the center of explosion the following approximate expressions are valid:

$$R = C_R\eta^m \quad N = C_N\eta^{-m} \text{ at } \eta \to 0 \quad \chi = 0$$

where

$$m = \frac{\nu - \kappa\omega}{\kappa - 1} \tag{19}$$

The matching conditions for N or R provide a relation between α and β, for example,

$$\tilde{N}_{\tilde{\eta} - \infty} = N_{\eta - 0} \cdot \chi^\alpha = C_N\eta^{-m}\chi^\alpha = C_N\tilde{\eta}^{-m}\chi^{\alpha - m\beta}$$

from which it follows that

$$\alpha = m\beta = \frac{\nu - \kappa\omega}{\kappa - 1}\beta \tag{20}$$

From Eqs. (18) and (20) we find

$$\alpha = \frac{\nu - \kappa\omega}{2(\kappa - 1) + (\nu - \kappa\omega)(n + 1)}$$
$$\beta = \frac{\kappa - 1}{2(\kappa - 1) + (\nu - \kappa\omega)(n + 1)} \tag{21}$$

The equations of the central boundary layer are derived in just the same way as for the frontal layer. We get

$$\tilde{R}\tilde{V}' + (\tilde{V} - \tilde{\eta})\tilde{R}' + \alpha[\omega(1 - 1/\alpha) - 1$$
$$+ \tfrac{1}{2}(2n - 1)(\omega - \upsilon)]\tilde{R} + (\upsilon - 1)\tilde{R}\tilde{V}\tilde{\eta}^{-1} = 0 \tag{22}$$
$$\tilde{N}^n\tilde{N}' = \sigma\tilde{R}\tilde{N}(\tilde{V} - (1/\kappa)\tilde{\eta}) \qquad (\tilde{R}\tilde{N})' = 0$$

Naturally, the prime in Eqs. (22) refers to differentiation with respect to $\tilde{\eta}$.

The boundary conditions for the solution of the problem of the central boundary layer are obtained from Eqs. (12) and from the conditions of matching with the outer solution. They have the form

$$\tilde{V}(0) = 0 \quad \tilde{V}(\infty) = (1/\kappa)\tilde{\eta} \quad \tilde{N}(\infty) = C_N\tilde{\eta}^{-m} \tag{23}$$

where m corresponds to Eq. (19), whereas C_N is found from the solution of the outer problem.

Some characteristic features of the central boundary layer are evident from the outward appearance of Eqs. (21) themselves. First, it is clear that, like an aerodynamic boundary layer, it possesses the property that the pressure is constant across the layer. Second, the order of the thickness of the layer is evident from Eqs. (17), and is equal to

$$\tilde{\delta} = O(\chi^\beta)$$

The quantity β, determined by one of Eqs. (21), is usually less than unity, and with $\omega = 0$ might be even of the order of 0.01. This means that, unlike the frontal layer, which has thickness $\delta_* = 0(\chi)$, the central boundary layer might be of a noticeable thickness, even at extremely small values of χ, when the quantity $\delta*$ may be regarded as negligibly small.

An investigation of the central boundary layer, for the particular case of an explosion with $n = 1$, $\nu = 3$, $\omega = 0$, and $\kappa = 1.4$, was conducted by Sychev.[3] Having as our goal a numerical investigation of the complete Navier-Stokes equations, we here consider the analysis of the two boundary layers as an intermediate stage through which it is necessary to pass when studying the behavior of the solution within the singular region of small values of χ.

Evolution of Dissipative Flow in Time

As is proved by the solutions of the boundary layers, the effect of increase of the "time-like" argument χ on the character of the flow does not depend on the form of the function $\chi(t)$. For any law of variation $\chi(t)$, increase in χ leads to a smearing out of the shock front, and to a smoothing out of the singularities that arise at the center in the case of adiabatic flow. Such behavior of the solution is in accord with physical notions of the effects of dissipation, and there is no reason to believe that the qualitative trends revealed at small χ will change at any finite value $\chi_* > 0$.

Because we want to determine the direction in which the dissipative flow will evolve with time, let us write down the expression for the derivative

$$\frac{d\chi}{dt} = \chi\frac{U}{r_f}K(\chi) = \chi\frac{U}{r_f}[\omega - 1 + (2n-1)Z] \tag{24}$$

In accordance with the definition of χ, we shall always have $\chi > 0$. If one excludes from consideration some special problems concerning convergent waves in gases, it is possible also to put $U > 0$, $r_f > 0$, which means that the sign of the derivative $d\chi/dt$ coincides with that of the function $K(\chi)$. The latter should be a function of fixed sign because otherwise the interdependence between χ and t will not be unique, and the introduction of χ as one of the arguments becomes senseless. If we assume a constant sign for $K(\chi)$, it is sufficient for a qualitative estimate of the character of evolution of the given flow to know the sign of the quantity $K(0)$, which corresponds to Eq. (10) with $Z(\chi)$ replaced by $Z(0) = Z_0$. The most interesting case for us, the blast problem, gives

$$K(0) = \omega - 1 + \tfrac{1}{2}(2n-1)(\omega - \nu) \tag{25}$$

Note that, if we limit ourselves to the consideration of transfer processes on only the classical molecular level, then

$$\tfrac{1}{2} \leq n \leq 1$$

and, as follows from Eq. (25), an explosion in a homogeneous medium ($\omega = 0$) always gives $K(0) < 0$. This is just the case corresponding to the trivial common notion that viscosity and heat conductivity affect the blast wave propagation only within a certain, very short, initial period of time. However, the situation might be altered if the explosion occurs in a medium of variable density, corresponding to the law of Eq. (2). It is not difficult to see that the condition $d\chi/dt < 0$ fails if we have

$$\omega \geq \frac{2 + \nu(2n-1)}{2n+1} \tag{26}$$

where the case of equality corresponds to a modified similarity, existing when both the viscosity and heat conductivity are taken into account, i.e., applying to the complete Navier-Stokes equations [Eqs. (1)]. One case of a similar solution was considered in Ref. 13.

If the condition of Eq. (26) holds as an inequality, then the evolution of the gas motion produced by the blast proceeds in the reverse direction, from the commonsense viewpoint: the shock wave exists only within an initial time interval, whereas subsequently, the flow pattern becomes more and more "dissipative" in the sense that the zones of adiabatic flow vanish and the perturbed region, as a whole, is controlled by the Navier-Stokes equations only.

Numerical Calculations and their Results

The qualitative investigation carried out in the preceding involved certain assumptions of which the rigorous proof would be too cumbersome. Our belief is, however, that there is no necessity for such a proof if our ultimate goal consists in carrying out numerical computations. The additional analytical work is needed only in the case when one or another of the initial assumptions proves to be unjustified.

If the problem is of such a physical character that it might be governed by Eq. (11), construction of its solution can be carried out in the following steps:

1) The solution is assumed completely known at $\chi = 0$, and corresponds to a self-similar solution for adiabatic flow.

2) A uniformly valid solution is formed within a certain range of small values $\chi < \chi_*$ as a combination of two boundary layers matched with an outer region.

3) At the fixed given value $\chi = \chi_n \geq \chi_*$, a certain quantity $Z_n^{(0)} = Z_0(\chi_n)$ is chosen, and the mathematical problem is solved with the boundary conditions of Eq. (12). During the solution of the boundary-value problem, the quantity $\psi^{(0)}(\chi_n)$ is determined in accordance with Eq. (8), and the satisfaction of condition Eq. (9) is checked. If for the chosen $Z_n^{(0)}$ the condition Eq. (9) is not satisfied, then the values $\psi(\chi_{n-1})$ and $\psi^{(0)}(\chi_n)$ are used to obtain a new $Z_n^{(1)}$, and the solution of the boundary-value problem is built up anew. The iterations are repeated until we have satisfied the condition $|Z_n^{(i+1)} - Z_n^{(i)}| \leq \epsilon$, where ϵ is a given small quantity.

While proceeding in accordance with the scheme just outlined, we obtain at each χ_n the values of all the unknown quantities within the interval $0 \leq \eta \leq 1$; the totality of values Z_n represents the function $Z(\chi)$. When the quantities that we are interested in have been determined as functions of η and χ, it is necessary to find the relation between χ and t. Prior to that we shall determine r_f and U as functions of χ. From the definition of the quantity U, we have

$$\frac{dr_f}{dt} \equiv U = \frac{dr_f}{d\chi}\chi\frac{U}{r_f}K(\chi)$$

from which follows

$$r_f = r_{f*}\exp\int_{\chi_*}^{\chi} K^{-1}\chi^{-1}d\chi \tag{27}$$

It is assumed here that the value r_{f*}, which corresponds to a small value of χ_*, might be found with the help of information on the self-similar flow, just as are the values t_* and U_*.

We find, further,

$$\frac{dU}{dt} = \frac{dU}{d\chi}\chi\frac{U}{r_f}K(\chi)$$

and from this, with the help of Eq. (7), we obtain

$$Z(\chi) = \chi K(\chi)\frac{d\ln U}{d\chi}$$

As in the case of expression (27), it is found that

$$U = U_*\exp\int_{\chi_*}^{\chi} ZK^{-1}\chi^{-1}d\chi \tag{28}$$

Introducing the notation

$$\delta = 2/\nu + 2 - \omega$$

and taking into account that

$$U_*/r_{f*} = \delta/t_*$$

one could obtain, with the help of Eq. (24),

$$\frac{t}{t_*} = \frac{1}{2}(\nu+2-\omega)\int_{\chi_*}^{\chi}\left[\chi^{-1}K^{-1}\exp\int_{\chi^*}^{\chi}(1-Z)\,\chi_1^{-1}K^{-1}d\chi_1\right]d\chi + 1 \qquad (29)$$

As an example, for numerical calculation we took the case of a point explosion with the following set of parameters: $\nu=3$, $\omega=2$, $\kappa=1.4$, $n=0.5$, $\sigma=0.7$. In order to avoid the troubles connected with severe limitations on the step size in χ, preference was given to an implicit scheme. With that scheme, at each step in χ and within the limits of each iteration, we are to solve a two-point boundary-value problem. In view of the existence of singularity along the line $\eta = 1$, at any finite value of χ, the asymptotic expansions must be used; namely,

$$N=A_N(1-\eta)^2+\tfrac{2}{3}A_N(Z+1)(1-\eta)^3+\ldots$$

$$V=3A_N(1-\eta)^2+C(1-\eta)^3-A_N(Z+7)(1-\eta)^3\ln(1-\eta)+\ldots \qquad (30)$$

$$R=1+2(1-\eta)+3(A_N+1)(1-\eta)^2+C(1-\eta)^3+(10A_N+4)(1-\eta)^3-A_N(Z+7)(1-\eta)^3\ln(1-\eta)+\ldots \quad A_N=1/(4\chi)^2$$

In Eqs. (30) the constant coefficient C cannot be found solely on the basis of conditions at the point $\eta=1$. Its value is set initially at random, to be defined more precisely later, in the course of the solution of the boundary-value problem at the given χ.

The results of calculations are shown in the figures, the transition from the value of χ to that of the ratio t/t_* being carried out in accordance with Eq. (29). In Fig. 1 is shown the variation of the dimensionless gas velocity V with $\eta=r/r_f$ for several values of t/t_*. The corresponding curves for dimensionless temperature are shown in Fig. 2, and for dimensionless density in Fig. 3. When analyzing the pattern of these curves, one must keep in mind that the scales of the hydrodynamic parameters [Eq. (6)] themselves depend on t/t_* by virtue of Eqs. (27) and (28). The dependence $r_f(t)$, defined by Eqs. (27) and (29), is shown graphically in Fig. 4.

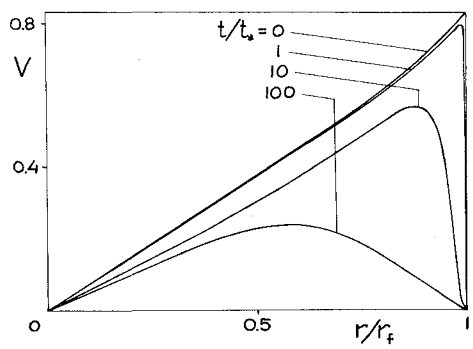

Fig. 1 Dimensionless velocity of gas vs distance from the center.

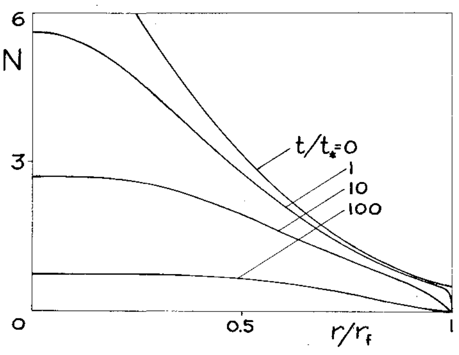

Fig. 2 Dimensionless temperature of gas vs distance from the center.

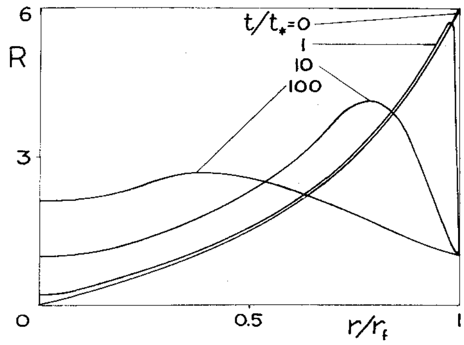

Fig. 3 Dimensionless density of gas vs distance from the center.

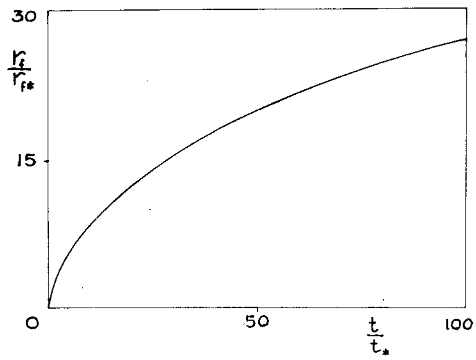

Fig. 4 Relative distance from the center to the front vs time.

The illustrated results give full confirmation of the qualitative conclusions stated previously concerning the effect of dissipative phenomena on the evolution of shock waves. We must draw the reader's attention to a specific feature that was not mentioned earlier by the authors of similar investigations. The quantity χ serves as a main criterion for the manifestation of dissipation effects and, other things being equal, χ is inversely proportional to the density of the gas at the perturbation front [see Eqs. (5)]. For cases when $d\chi/dt<0$, it follows that the influence of dissipation is more essential, when the gas is more rarefied in its initial state. But when $d\chi/dt>0$, then, independently of the initial density of the medium, sooner or later a time will come beyond which the process of motion could no longer be considered without taking into account viscosity and heat conductivity.

References

[1] Sedov, L. I., *Similarity and Dimensional Methods in Mechanics*, Academic Press, New York, 1959.

[2] Korobeinikov, V. P., "On Propagation of the Strong Spherical Blast Wave in Heat Conducting Gas," *Akademiia Nauk SSSR Doklady*, Vol. 113, 1957, p. 1006 (in Russian).

[3] Sychev, V. V., "On the Theory of Strong Explosion in Heat-Conducting Gas," *Applied Mathematics and Mechanics*, Vol. 29, 1965, pp. 1177-1184; transl. of Prikladania Matematika i Mekhanika, pp. 997-1003.

[4] Bowen, J. R. and Feay, B. A., "A Refinement of the Cylindrical Blast Wave," *Astronautica Acta*, Vol. 15, 1970, p. 275.

[5] Terent'ev, E. D., "The Structure of Shock Waves in Hypersonic Flows," *Applied Mathematics and Mechanics*, Vol. 38, 1974, pp. 227-237; transl. of pp. 254-263.

[6] Van Dyke, M., *Perturbation Methods in Fluid Mechanics*, Academic Press, New York, 1964.

[7]Becker, R., "Stosswelle and Detonation," *Zeitschrift für Physik*, Vol. 8, 1921, p. 321.

[8]Thomas, L. H., "Note on Becker's Theory of the Shock Front," *Journal of Chemical Physics*, Vol. 12, 1944, p. 449.

[9]Morduchow, M. and Libby, P. A., "On a Complete Solution of the One-Dimensional Flow Equations of a Viscous, Heat-Conducting Compressible Gas," *Journal of the Aeronautical Sciences*, Vol. 16, 1949, p. 674.

[10]Gilbarg, D. and Paolucci, D., "The Structure of Shock Waves in Continuum Theory of Fluids," *Journal of Rat. Mechanics and Analysis*, Vol. 2, 1953, p. 617.

[11]Sychev, V. V., "On Hypersonic Flows of Viscous Heat-Conducting Gases," *Applied Mathematics and Mechanics*, Vo. 25, 1961, pp. 898-912; transl. of pp. 600-610.

[12]Shakhov, E. M., "The Flow of Viscous, Heat-Conducting Gas within Hypersonic Shock Wave," *Inzhenerny Zhournal*, Vol. 4, 1964, p. 646 (in Russian).

[13]Shidlovskii, V. P., "Self-Similar Motions of a Viscous and Heat-Conducting Gas Resulting from a Sudden Deposition of Energy," *Fluid Dynamics*, Vol. 7, 1972, pp. 466-472; transl. of *Mekhanika Zhidkosti i Gaza*, Vol. 3, p. 117; also ASME publ. 71-WA/HT-3, 1971.